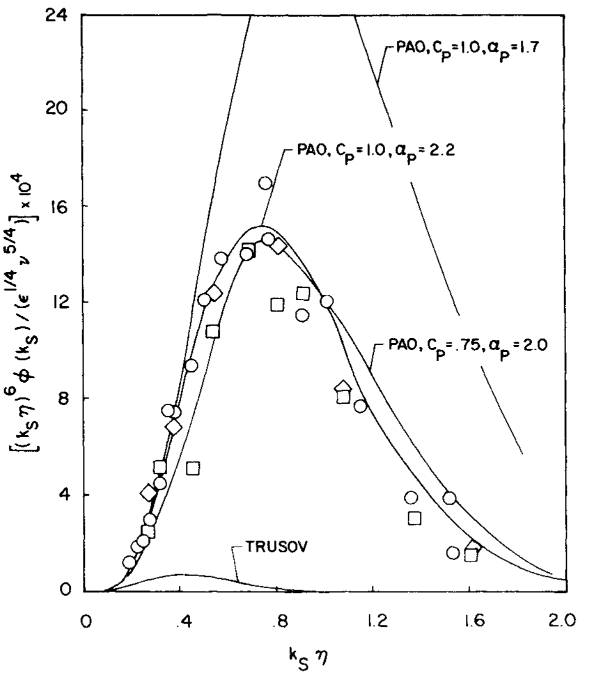

Fig. 2 Variation of the spectrum function of $\partial^3 u_1/\partial x_1{}^3$ fluctuations with wave number.

the derived results do not match the test results at the higher values of $k_S\eta$. Using $C_P = 0.75$ and $\alpha_P = 2.0$ gives fair agreement while giving the desired limiting value of $A(R)$.

The one-dimensional energy spectrum functions for Trusov's theory and Pao's theory with $C_P = 0.75$ and $\alpha_P = 2.0$ are compared with the test data of Ref. 5, represented by Pao's results for $C_P = 1.0$ and $\alpha_P = 1.7$, in Fig. 3. Trusov's theory again shows differences from the experimental data.

Based on these results it is felt that the use of Pao's density function with $C_P = 0.75$ and $\alpha_P = 2.0$ provides the best over-all

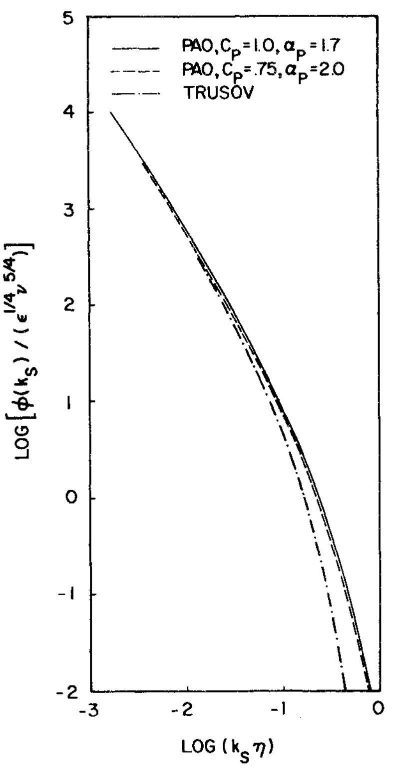

Fig. 3 Variations of the spectrum function of u_1 fluctuations with wave number.

match with the various forms of the available test data. Considering the variation of $A(R)$ with Re obtained with this density function, the following relations are obtained:

$$A(R) = 0.16 + 3.54/Re, \quad \text{for} \quad Re \geqq 1.0$$

$$A(R) = 0.16 + 3.54/(Re)^2, \quad \text{for} \quad 0.5 < Re \leqq 1.0$$

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Edge Restraint Effect on Vibration of Curved Panels

L. W. Rehfield,* J. Giri,† and C. A. Sparrow‡
Georgia Institute of Technology, Atlanta, Ga.

Introduction

In recent years, investigations have shown that edge restraint effects are quite important in the buckling of shell structures. The compressive buckling of cylindrical panels with various limiting cases of edge restraint along the unloaded edges has been studied by Rehfield and Hallauer.[1] A less exhaustive study of cylindrical panels under external pressure has been done by Singer et al.[2] These results indicate a high degree of edge restraint sensitivity for buckling and prompt us to investigate this effect on vibration in this Note.

Some attention has been paid to edge restraint effects on vibration. Forsberg[3] studied complete homogeneous isotropic cylindrical shells and found a particularly pronounced effect

Received July 13, 1973. This work was supported by the U.S. Air Force Office of Scientific Research under Grant AFOSR-73-2479.

Index category: Dynamic Structural Analysis.

* Associate Professor of Aerospace Engineering. Member AIAA.

† Graduate Research Assistant.

‡ Research Engineer, Rich Electronic Computer Center.

due to restraint of the axial (longitudinal) displacement component. Palmer,[4] Sewall,[5] and Webster[6] have studied the vibration of cylindrical panels with one type of simple support along the edges and with completely clamped edges; for the most part, these authors present approximate solutions based upon Rayleigh or Rayleigh-Ritz methods.

A rather complete study of the influence of edge restraint on simply supported cylindrical panels has been reported by Matsuzaki.[7] Various types of in-plane or tangential boundary conditions on both the curved and straight edges have been considered and an over-all evaluation of their effect can be made. If the ratio of the curved edge length to the straight edge length is smaller than unity, boundary conditions on the curved edges are very influential; if this ratio is larger than unity, the conditions on the straight edges are more influential. In the latter case, the vibration results for the fundamental mode are qualitatively similar to those found for compressive buckling by Rehfield and Hallauer for both simply supported and clamped edges.

In this Note, we present rigorous solutions for square panels vibrating in their fundamental mode. Both simply supported and clamped edges with various limiting types of tangential restraint along the straight edges are considered. The simply supported solutions agree with the approximate solutions of Matsuzaki obtained by Galerkin's method. Three of the four clamped solutions are believed to be new and provide sufficient information for a comparison with the simply supported cases.

Basic Theory

An isotropic, Hookean elastic cylindrical panel with radius of curvature R, developed width b, length a, and thickness t is shown in Fig. 1. The origin of surface coordinates (X, Y) is the center of the panel, and U, V, and W are midsurface displacement components.

The analysis is based upon the following equations, which are equivalent to those of Donnell[8] for lateral free vibration

$$\nabla^4 f + 2Kw,_{xx} = 0 \tag{1}$$

$$-2Kf,_{xx} + \nabla^4 w - 4\lambda w = 0 \tag{2}$$

$$u,_x = f,_{yy} - \nu f,_{xx} \tag{3}$$

$$v,_y = f,_{xx} - \nu f,_{yy} + 2Kw \tag{4}$$

The dimensionless variables and parameters are listed below, along with other important quantities

$$x = \pi X/b \qquad y = \pi Y/b$$

$$K = \{[3(1-\nu^2)]^{1/2}/\pi^2\}(b^2/Rt) = \sigma_c/\sigma_p$$

$$w = [12(1-\nu^2)]^{1/2} W/t \qquad \{u, v\} = (4\pi E/\sigma_p b)\{U, V\}$$

$$\sigma_p = [E/3(1-\nu^2)](\pi t/b)^2 \qquad \sigma_c = \{E/[3(1-\nu^2)]^{1/2}\}(t/R)$$

$$\lambda = (\omega/\omega_{sp})^2 \qquad \omega_{sp}{}^2 = [Et^2/3(1-\nu^2)\rho](\pi/b)^4$$

$$\nabla^2 = [\partial^2(\ )/\partial x^2] + [\partial^2(\ )/\partial y^2] \qquad \nabla^4(\ ) = \nabla^2[\nabla^2(\ )] \tag{5}$$

The stress parameters σ_p and σ_c are the compressive buckling stresses of long or square simply supported plates and one type of simply supported cylindrical shell, respectively; ω_{sp} is the fundamental frequency of a square simply supported plate; E is Young's modulus; ν is Poisson's ratio; and ρ is the mass density of the plate material. f is a dimensionless Airy stress function defined such that the membrane stresses, $\sigma_x{}^o$, $\sigma_y{}^o$ and $\tau_{xy}{}^o$, are given by the relations

$$\{\sigma_x{}^o, \sigma_y{}^o, \tau_{xy}{}^o\} = (\sigma_p/4)\{f,_{yy}, f,_{xx}, -f,_{xy}\} \tag{6}$$

Harmonic motion with circular frequency ω has been assumed, and the effects of in-plane or tangential inertia have been neglected.[9]

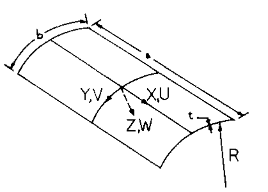


Fig. 1 Circular cylindrical shell coordinate system and panel geometry.

The effect of various types of restraint applied to the straight edges ($Y = \pm b/2$) is the problem considered here, and, for this purpose, it is assumed that the restraint applied to both edges is identical and that the vibration modes are sinusoidal functions of X. This latter assumption implies that

$$w = w,_{xx} = \sigma_x{}^o = v = 0 \tag{7}$$

at the edges $X - \pm a/2$; this is the $S_x 2$ type of restraint along the curved edges discussed by Matsuzaki.[7]

Limiting cases of edge restraint which correspond to the vanishing of generalized forces and generalized displacements at a straight edge can be separated into two categories—normal and tangential. Normal edge conditions characterize the bending and twisting of the edge, whereas tangential (in-plane) edge conditions characterize the stretching of the panel midsurface. The following scheme of classification, due to Rehfield and Hallauer,[1] is adopted. For normal edge conditions ($Y = \pm b/2$), SS is the simply supported class for which

$$w = w,_{yy} = 0 \tag{8}$$

CC is the clamped class for which

$$w = w,_y = 0 \tag{9}$$

For tangential edge conditions ($Y = \pm b/2$)

$$1)\quad \sigma_y{}^o = \tau_{xy}{}^o = 0 \tag{10}$$

$$2)\quad \sigma_y{}^o = u = 0 \tag{11}$$

$$3)\quad v = \tau_{xy}{}^o = 0 \tag{12}$$

$$4)\quad v = u = 0 \tag{13}$$

A complete characterization of the edge restraint is obtained by selecting one normal and one tangential edge condition. Thus, the designation SS2 implies that Eqs. (8) and (11) are enforced along the straight panel edges.

Solutions to Eqs. (1) and (2) of the appropriate type are found by assuming

$$\{f, w\} = \{A, B\}\, e^{py} \cos mx \tag{14}$$

A and B are constants and m is a wavelength parameter which will be called the aspect ratio of the vibration mode. If M denotes the number of half-waves that occur in length a, then

$$m = b/(a/M) \tag{15}$$

If Eqs. (14) are substituted into Eqs. (1) and (2), closed-form expressions for the eight roots p can easily be found as functions of λ, m and K. The exact nature of the roots depends in a complicated way on λ, m and K, so all calculations have been programed in complex arithmetic for the Univac 1108.

With the differential equations satisfied, the usual vibration determinants, which correspond to nontrivial satisfaction of prescribed boundary conditions, can be formulated and solved for the values of λ corresponding to free vibration for fixed values of m and K. The details of the calculations are straightforward and will not be discussed.

Results and Discussion

All calculations have been carried out for square panels ($m = 1$) and attention has been restricted to the fundamental modes of vibration in all cases. The effect of edge restraint is expected to diminish for the higher modes.[7]

The results for simply supported panels are shown in Fig. 2. The SS2 solution corresponds to the double-cosine mode of vibration

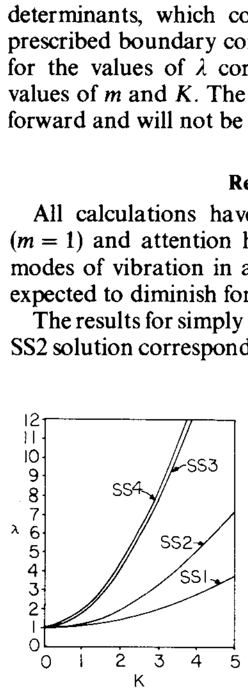
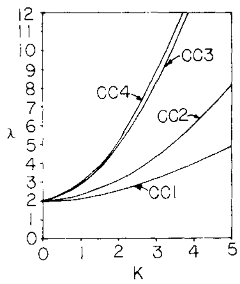


Fig. 2 Results for the fundamental mode of simply supported panels ($m = 1$).

Fig. 3 **Results for the fundamental mode of clamped panels ($m = 1$).**



which has been studied extensively.[3-7] The results for the other cases agree with the approximate solutions of Matsuzaki.[7]

The results for clamped panels are shown in Fig. 3. The CC4 case has been studied previously[4-6] by approximate methods. The remaining three solutions are believed to be new, and they permit a comparison to be made with the simply supported cases.

As can be seen from Figs. 2 and 3, the effect of tangential boundary conditions is similar for simply supported and clamped edges. Furthermore, the trends indicated are similar to those found in the compressive buckling cases by Rehfield and Hallauer. We conclude, therefore, that the fundamental vibration frequency is sensitive to rotational restraint ($w,_{yy} = 0$ or $w,_y = 0$), circumferential restraint ($\sigma_y{}^o = 0$ or $v = 0$), and longitudinal restraint ($\tau_{xy}{}^o = 0$ or $u = 0$) of the straight edges, in decreasing order of influence.

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Pressure Sources for a Wave Model of Jet Noise

Y. Y. CHAN*
National Research Council of Canada, Ottawa, Canada

THE structure of the turbulent jet has in recent years been studied extensively in connection with noise generation. It was found that the structure of the turbulent mixing region is organized fairly orderly and behaves like a wave train similar to the hydrodynamic stability waves propagating in a shear flow. Among many other experimental observations, Mollø-Christensen[1] and Crow and Champagne[2] gave the most interesting accounts of this wavelike phenomenon. A detailed measurement of the spatial growth and decay of the pressure wave in the turbulent mixing layer of a low-speed jet has recently been reported by Chan.[3] This regular structure in the turbulent mixing region has also led to some revision of the turbulence models used in the jet noise computations. In Michalke's model[4,5] the wave character of the flow is incorporated explicitly. The source function is first resolved into Fourier components in the azimuth angle and for each frequency the source component is assumed to be wavelike. The far field solutions thus obtained generate all essential features of the sound pressure distributions observed experimentally. In Michalke's work, the exact form of the source components are not yet known and in order to illustrate his solutions, several forms of source distributions are assumed. The source term in Michalke's wave model, however, can be readily written in terms of the fluctuation pressure according to the dilation theory of Ribner.[6] The experimental results obtained by Chan[3] for the pressure wave development in the mixing region can then be used to evaluate the proper form for the source function.

From the dilation theory, the source term, q, can be written as

$$q = (1/a_0{}^2)\,\partial^2 p^{(o)}/\partial t^2 \tag{1}$$

where a_0 is the speed of sound of the ambient air and $p^{(o)}$ the pressure fluctuation in the jet. The single frequency component of the source term can be obtained by Fourier transform of Eq. (1). Thus the mth azimuth Fourier component for a single frequency can be written as

$$Q_{m\omega}(x, r) = -k^2 P_{m\omega}{}^{(o)}(x, r) \tag{2}$$

where $k = \omega/a_0$ is the wave number of the sound wave at the frequency ω. Assuming the pressure source $P_{m\omega}{}^{(o)}(x, r)$ is separable in the independent variable x and r

$$P_{m\omega}{}^{(o)}(x, r) = F(x)G(r) \tag{3}$$

where $F(x)$ and $G(r)$ are, respectively, the longitudinal and lateral amplitude distributions of the pressure waves inside the jet. Then it can be shown that the far field sound pressure, $P_{m\omega}$, is proportional to the source integrals[4]

$$P_{m\omega} \propto \frac{R^2 L}{2} \int_0^1 \int_0^1 \hat{F}(\bar{x})\hat{G}(\bar{r})\bar{r}\, d\bar{r}\, d\bar{x} \cdot I_r \cdot I_x \tag{4}$$

where

$$I_r = \frac{\int_0^1 \hat{G}(\bar{r})\bar{r} J_m(kR \sin\theta\bar{r})\, d\bar{r}}{\int_0^1 \hat{G}(\bar{r})\bar{r}\, d\bar{r}}$$

$$I_x = \frac{\int_0^1 \hat{F}(\bar{x}) \exp\left[i\alpha L(1 - M_c \cos\theta)\bar{x}\right] d\bar{x}}{\int_0^1 \hat{F}(\bar{x})\, d\bar{x}}$$

and

$$\hat{F} = F/F_{\max}, \quad \hat{G} = G/G_{\max}, \quad \bar{x} = x/L, \quad \bar{r} = r/R$$

L and R are the extents of the source region in the x and r directions, respectively. θ is the angle in polar coordinates for the measuring point and Jm the Bessel function of order m. The convection Mach number is the ratio of the phase velocity c_{ph} of the pressure wave and the ambient sound speed a_0. The phase velocity of the pressure wave is given in Ref. 3.

The experimental data of Ref. 3 show that the longitudinal distributions of the amplitude of the pressure waves for different Strouhal numbers are very similar to each other when the data are plotted against a normalized scale of distance $St\, x/D$, where St is the Strouhal number of the jet based on the jet velocity U

Received July 30, 1973; revision received September 10, 1973.
Index categories: Aircraft Noise, Aerodynamics; Jets, Wakes and Viscid-Inviscid Flow Interactions.

* Associate Research Officer, High Speed Aerodynamics Section. Member AIAA.